Good afternoon. So last Friday I was on a trip. RPI school closed anyway. So in announcement I think I signed on Saturday. And I asked you to review lecture 12. So I asked you to watch the video I recorded a year ago. And that lecture is about actually physics, the model, actually source and the detector. So those things are needed to understand the CT scanner imaging system. And I know some of you haven't watched the video yet. But pay attention to this lecture. You can still follow once you understand the X-ray measures line integral. If you know that, you can still follow this lecture. Pretty much focused on image reconstruction. But you do need to review the textbook and the previous recording. Okay. Let me get the visual pointer. Okay. Very good. Very good. Okay. And then we finish the first examination. And the average score isn't perfect. But that's the purpose. We don't want the overall score distribution to focus towards the high end. So this is something you have good discriminative power and see your understanding. Some questions are tricky. And I ask you to understand the material better. You better follow the format. You need to do preview and the lesson in the lecture and do review. And I underline the fundamental part like Fourier analysis, sampling, theorem. You need to take a major effort. And then in the modality part, like later on, we explain MRI pulse sequences. And those things, and the CT, today we explain filtered bag projection algorithms. So these things are by no means easy. So you need to read the lesson, review, do homework, and then ask the question. And then I decide I will be responsive. So you send me emails. So far I reply quickly. And the TA has been very helpful. And I think we will have a spring break soon. So over spring break, I should be able to finish revision of the foundational part, the part one. So make things streamlined because what you show is rough draft. But the next one is going to be pretty much presentable. And also, imaging modality-wise, we have the green textbook. And I asked the TA to upload relevant chapters you can read. The green textbook is good, some content out of date. So you can read the green textbook and cover majority of the technical content. And you pretty much know the basic idea. And in the lecture, I will explain some essential concepts so you have a little bit insight. So this is the outline for today's X3 CT reconstruction lecture. So first, let me give you some very rough idea. Once you know the idea, you will be in a good position to understand the two main approaches for image reconstruction. The first type of algorithms, or we call it algebraic approach, we treat images as unknown and we try to establish the system of linear equations. The next one is analytic approach. So we pretty much rely on Fourier transform. So you will see why I have been emphasizing the Fourier analysis. So if you understand that part very well, you will have deep insight that you see why you can use Fourier analysis to do image reconstruction. So before I go into the two approaches, let me just say a little bit to underline the information. Like in physics, you have particle property and you have wave property. So for image reconstruction, we can kind of make an analogy. We say underlying image, we can view underlying image in two ways. The first way is that we view the image as a collection of pixels if the image is two-dimensional, or you view it as pixels for three-dimensional images, a collection of particles. Really here we just say picture elements, the pixels, the pixels. And the other way is complementary. We view image as a superposition of waves. So for two-dimensional image, for example, we think of the image as just a summation of all kinds of waves propagating along different orientations. And for a given orientation, the wave can be at different frequencies, and you have amplitude, frequency, phase. But if you have all these parameters set up right, you add all these waves together, then you have the images. So this is the basic idea. So you have this high-level idea, particle and wave perspective of image and image reconstruction. So let me explain a little bit further. So you have arbitrary picture, our former president. So you have the picture. Then you can really decompose the picture into many, many small elements. And for each element, it's very simple. Just a pixel homogeneous. It's just a small square. So you add these together. The trick is really amplitude. You just need to make sure all these amplitudes are modulated nicely. Then put it together, you have perception of a picture or natural scene whatsoever. So this is one way to represent a picture. Another way, I mentioned a Fourier analysis. So in a two-dimensional picture, you perform a Fourier analysis. You have a Fourier spectrum. You arbitrarily pick one. So you have the frequency proportional to the distance from the origin to the given point in the Fourier domain. So this is an example. So you have a frequency shown here, and then you have amplitude which is proportional to the value at this particular point. So you have this particular wave propagating along that direction. And the Fourier analysis really utilizes all kinds of wave components. And you have DC components. You have horizontal wave. You have vertical wave. Low frequency, high frequency, and the wave can also propagate in arbitrary directions. So all these kind of waves. And the trick is the setting of the parameter. And once you find the coefficient, you add the Fourier component together. And then you can recover the image. In this case, Albert Einstein. So there's two ways. And based on these two perspectives, we can have two approaches. And I mentioned algebraic and analytic. So let me first explain the first approach. So actually measurement can be put in line integral or resum format. So let me explain. So first you have incoming intensity. You have output intensity. So output is related to input and subject to linear attenuation. So for single element, this will be simply the mu times delta

x. Delta x is this pixel size. You have multiple material components. So basically you have
incoming intensity attenuated by first pixel. The output of first pixel is input to the second pixel,
so on and so forth. So you add all these things together. You have this relationship.
This kind of review of previous lecture. You really should watch that lecture. So output attenuated
quantity is input multiplied by this exponential factor. And in the index part, you have summation
of all the little mu k weighted by delta x. So this is the imaging model or data model.
So input quantity you know and you can measure. That's the flux of your actual tube. Output you
can record with your actual detector. We explained in the previous lecture what is tube, what is
detector, how they work. We got this relationship. So what do you know? You know incoming intensity.
You know the detected attenuated intensity. And this delta x is what you specified. You
know delta x. What you don't know? You don't know all these mu k. k equal to 1, 2, 3 until
some number n. So if you do normalization, so you do this n i divided by n o output
and just move this the other part, you do natural log. So you got this. So this is just a result
going from the Beer's law and to this reformated or normalized form. So you see this is nothing
but a linear system equation because the mu k is unknown and delta x is a weighting factor.
And this n i, n o, all known quantities, you do log, you still know. So this is nothing but
linear equation. So for each measurement along x-ray path, you have one linear equation. You do
many, many measurements. You have a bunch of linear equations. So you have a system of linear
equations. You can solve equations. And now we treat image as pixels and we interpret x-ray
measurement as linear equation. So many, many measurements. You have a big system of linear
equation. So this is algebraic perspective to solve tomographic problem as a solution to the linear
system equation. And if you take limit, so you make the pixel very small, so the summation
becomes integral. So you have this integral. So still right-hand side normalized and the natural
log on the right-hand side. So the discrete format, we call it resum because this is summation.
And here is a line integral because this is integral along the line which is x-ray path.
So this is data. So x-ray measurement gives data as resum or line integrals. It depends on you want
to view it in discrete format or in continuous domain. So this is a model. If you haven't
reviewed the previous lecture yet, just follow this. So you understand this. Okay, follow this.
Now let me explain further. So you have all these resum measurement and you can solve linear system
equation. And why you can solve linear system equation so that you can have unique solution,
let me give you some heuristic explanation. And this is what I call unimpeeling idea.
So just think you have a picture and you decompose the picture into collection of pixels.

The pixel is not necessarily rectangular. So here I use triangular pixel. So you just see
I do the decomposition. So into many, many triangle pixels. Then I just try to resolve
the unknown layer by layer. Just see the outermost unknowns. So it's represented as a right triangle.
This is a heuristic idea. So try to follow. So in limiting case, you think you have an
object support. The x-ray just touch one molecule. So it's a very small thing. So just take this as
example. You have a small material element. You know the incoming intensity. You know the
attenuated intensity. And because you data the partition, so you know this total length.
So from these three known, you can resolve the only unknown. This is a mu, say mu zero, mu one.
This is a linear coefficient of this pixel. And by assumption, this pixel is homogeneous.
So simple argument like that. Now you know the mu for this right triangle. And the same argument,
all these right pixels can be directly measured with a prior flow x-rays.
Once that, we can move to next layer. Next layer is a green one. So it's a green x-ray.
You know this incoming intensity. You know attenuated intensity. And now you know the mu
for this right element. And you also know mu here. Because you know mu, so you can compute
the incoming intensity, got attenuated. What's the incoming flux into this
green pixel? You know here. Then you know attenuated intensity here. You know this
attenuated intensity is the quantity just out of this green pixel, but attenuated by this
right pixel. And then you can use Beer's law in a reverse fashion. So from this attenuated value,
you know what's the value here. So you know value here, you know value here, you know total length.
So you can resolve the mu for this particular green triangular pixel. You follow me?
So likewise, so each of the green elements is resolved.
Okay. And then now you move to this light blue ray. And now the green and the right are known.
So only this one is known. So this argument, if you follow these heuristics, I put, you know,
tied at relation. So I explain this in this way without complicated mathematical argument.
This is heuristically. You see the picture. You can peel the amine layer by layer so you can resolve
the underlying image algebraically. So you try to solve linear system equation. Essentially,
you are doing things like this. So pixel-wise, so you do layer by layer. So you know you send
all the rays in parallel beam geometry. You send all the rays possible. You can always
resort, reorganize the x-rays in a way. So you can view it from outermost layer.
Then layer by layer, you resolve all the unknown. So this is a key argument. So do you follow me?
Understand? Any questions?
So this is a line integral. So along this ray, you know the incoming intensity.
You know this attenuated intensity. What you don't know? You don't know all these
So you understand this equation. So you know this is, you just, this is Beer's law. So from
Beer's law, you do the normalization. So this right-hand side is known quantity. So what you

don't know is the mu k. Delta x, you also know. So this is a line integral. So what you can measure
is nothing but a line integral. The line integral in this case, this special case,
you only have a single unknown mu. Single unknown mu, and then you know this incoming
and output intensity. And you can use this equation because you only have one mu unknown.
So you can solve this. You just recover this mu for this right pixel, right?
This is Beer's law. So once you resolve all these right pixels, for green pixel,
you can measure this, you can measure that. But now the right mu's are known. So you can compute
how strong the light, what's the flux into this green pixel. Because suppose you have
100 photons, you know mu, you know that the photon will be attenuated in the right pixel.
So we'll bring the number down from 100 photons to 60 photons. So you know 60 photons is injected
into the green pixel. So you have 10 pixels here. You know this 10 pixel at this point
is attenuated quantity by this green pixel. You know the right mu here. So you can say
this 10 pixel, really here you have 30 pixels. Because of this known mu, we have attenuation
process. So this 30 photons bring down to 10 photons. So just use Beer's law in reverse fashion.
Now also you know you have incoming photon number, you have attenuated photon number.
Based on that, certainly you know the path lines, you did the partition.
Then you can find the mu for this green pixel. Now you understand it's very heuristic. So layer
by layer, you just peel that in that way. So with this heuristic, this is my idea. I think this is a
cool explanation. So with this idea, you understand you have all kinds of X-rays
shooting through the cross-section. You have all the rays are useful. And each layer by layer,
each new layer from outermost towards the center, each layer bring you little new information.
With all of them, you happen to have enough information to fully resolve this cross-section.
This is why the system of linear equation is sufficient for you to recover underlying image.
You follow now? Okay, good. So mathematically, and we can say data sufficiency condition for
two-dimensional image reconstruction. So you have a cross-section. And you arbitrarily draw
three lines. So this is the X-ray path. You arbitrarily draw a line. Then we say you can
find at least one source position. What does that mean? That means along this line, X-ray source,
I made rays going through this direction. So this line integral or resum is measured.
It doesn't mean you have any information you need. So arbitrarily draw a line as long as the line
intersects with the cross-section. Then we say we have data. So that's a maximum amount of data
you could have. And that is sufficient according to this simple understanding. So this is
way and I understand the subject or I try to explain to student. I always want to give you
picture and some visualization geometrical ideas. And it could be easy. You're confused. You can
watch my lesson. So right now, I think about half a million viewers, a lot of light. So I really

hope your watch give me light. That's good. And some other heuristics, it's so important,
but it's not that easy. Like I spend a lot of time for analysis how you compute the coefficient.
I say the coefficient is nothing. It's an inner product. It's a high dimensional vector projected
onto basis function. That is a very important heuristic. But I doubt all of you understand.
If you don't, please review. And I hope by the end of this month, I will upload
newer version. That will be much better than the rough draft. So you read it. Even I wouldn't
test again, but you need to understand the Fourier analysis so that you understand
CT and MRI much better. But anyway, so this is the idea about data sufficient condition.
Okay. Then with X-rays, and we can measure parallel beam projection as shown here. So
along each X-ray and each datum, give you one linear system equation. So you have a parallel beam
at projection angle theta. Then you have, say, if you have 100 rays or 500 rays, you have
500 linear system equations. This one view wouldn't be enough. This one view is not enough
because you even cannot resolve. You see two things superimposed together. And which one on
top, which one beneath, you don't know. So you need to keep changing the theta. Then you have
an original function f of x, y converted to a new two-dimensional function p of theta t. Theta
is this angle. T is the coordinate system. Okay. So you just use X-ray measurement. You
did a physical transform or mathematical transform from f of x, y to p theta of t.
So something like this. You see you have a bright small disk. The bright small disk will
trace a sinusoidal curve. That's why we call the data domain representation
sinogram. And also we call it a radon transform. Radon is a mathematician many years ago.
So this is projection. This is one view. Sinogram, all the views are put together.
And the computed tomography, previously it is called the computer-aided tomography. So
C-A-T. So usually put a cute cat here. So C-T is nothing but the inverse process. Once you have
sinogram data, p of theta t. So given the data measurement, how you can invert the
process. So what is underlying image, you can interpret this data. Once you see the data,
you reconstruct the image. So just the inverse process is just from data to image. So X-ray
measurement is from image to data. And the tomographic algorithm is the other way, from
data to image, underlying image. And how you do it, so the picture I gave you shows you can use
an impeding method. So you have a heuristic feeling. Now let me give you a numerical example.
And in practice, the image can be 512 by 512. But for teaching purpose, let me just give you
two by two image. So this is a simple case. But the essential idea is already there.
Okay, you have pixel values, one, two, three, four. Very simple. But you don't know this.
This is just something I set up. What you are allowed to x-prone, you can use X-ray. And I

told you X-ray cannot pinpoint a single pixel. If you just have a magic pen, you just read out 0.1 pixel, you get a value out. Something like photograph. You don't need to do tomographic reconstruction. That's just too simple. So with X-ray measurement, we can get some information.
But you shoot X-ray this way. And I explained to you X-ray measurement.
And just to give you a re-sum. This is the earlier slide. So you do not send X-rays through these two pixels. And you will not be able to say what's mu1, what's mu2. But you do know what's the sum of these two pixels. And mu1 plus mu2 is 7. And from this, you do not still do not know mu1, mu2. But you know, OK, the sum is 7. So likewise, just for example, this is vertical rate.
So you got mu2, mu4. Mu2, mu4 added together, you have 4. So with X-ray measurement, you can bring up a number of equations. You solve the equation, you just get an unknown
That's the idea. So normally, you have an n by n image. So you have an n by n unknown. So here, you have 2 by 2, 4 unknowns. So you need to shoot 4 rays. You get 4 measurement. So 4 system, 4 linear equations, 4 unknowns. So it looks perfect, right? So not that simple. Look at this.
So you shoot these two rays. So you got this one. These two equations added together, right hand side will be 10. And the last two equations added together is still 10. So you just subtract one equation from the sum. Then you get the rest one. So it's a little tricky, but just say, I'm trying to say these four equations are not totally independent.
So really, from these three equations, you can derive the last one. So they are not totally independent. The trick, so the number of equations equal to the number of unknowns.
It's under the condition, each equation will give you some new information. So it will be independent. If not independent, you do not have enough number of equations to solve the problem.
So you really shoot a ray along this direction. So this diagonal direction. So you may get this one, this one, this one, plus this one. Then you got enough number of equations to solve uniquely.
But anyway, so remember, number of equations equal to number of unknowns is under assumption.
All these equations are independent. Otherwise, you do not have enough number of equations.
But for explanation, let me still use two horizontal rays, two vertical rays to show you how we solve linear system equations using so-called iterative algorithms through trial and error. Why I want to explain iterative algorithm? For a simple case like this, you can solve directly using the analytic scale. But when the number of unknowns is huge, like a million billion equations, and the direct method cannot work efficiently. You do not have computer memory and many problems. So you have to use iterative algorithm to solve the situation.
Also, iterative algorithm, I'm going to explain here, allows you to impose prior knowledge, like non-negativity or smoothness. So these topics are beyond the scope of this

lecture.
Let me just give you a sense of idea, how you can do trial and error to solve system of linear equations. As I told you, this is your underlying image. You have
four measurements, two horizontal, two vertical. You try to solve this system.
The starting point, whenever you use iterative algorithm, you need to have a starting point.
Here, our starting point is because I know nothing about images. So just to be neutral,
I just assume nothing in the field of view. So every pixel is zero. This is my starting point.
This is my starting point, or I call it guess zero. This is the natural and unbiased
starting point. First, let me say, if this is correct, then the vertical integral
must be zero and zero. This is my assumption, based on the assumption that I got,
these two estimated values, zero, zero. Then we call this predicted or synthetic projection.
It's not something, anything I say is zero, zero, you would accept. You can challenge me.
You say this is all zero, then vertical integral must be zero, zero. But the physical measurement
says this is six, four. It's not zero, zero. So how would you explain the contradiction?
So I just do the comparison between measurement and prediction. I see
error, six, four, because here you got six, here you got zero. So error is six. Likewise,
error is four. This positive error indicates that my data initial guess. So I underestimate
pixel value. So this is a problem. So along this, real measurement is four, but I just say zero,
zero. Clearly, this would be something more than zero, zero. So if something here, something
something in this pixel, something in this pixel, they add it together, should be six.
So six is the error here. I need to redistribute the error back. And I do not know if I should
give more contribution to first pixel or second pixel. So to be fair, I just evenly divide the
error into three, three. So I put the error back. Once I do this redistribution, I made sure they
added together will be six. So if they have three, three here, then this vertical error,
the error six, will be totally removed. So I am improving my solution. Likewise, this is four.
I just redistributed back to two. So once I finish this, so I am vertically happy. I mean,
so if this is our current result, I see no contradiction in terms of vertical projection.
So I got this six, two plus two is four. It's the same thing as what you measured. So I'm happy there.
But then you can go step further, challenge me, wait a minute, let's do, let's double check
horizontal integral. So horizontal integral, here you got five, you got five. What's the measurement?
Here you got seven. Here you got three. You do comparison again. You see the error two is error
minus two. You see positive error, that means along this row, you still underestimate what's
going on in the field of view. And the negative one means that the real thing is really less than
this. So again, I redistributed the error. This two is decomposing into one plus one. I did it back

and this decomposing into minus one plus minus one, redistributed back. In this case,
we are lucky. This is the two eight reason we got the correct results. Once you reach this status,
so vertical data, horizontal data, all perfectly explained. So we are done. It's just the idea.
But in real situation, never simple like this. You need to do many eight reasons and many unknowns
goes back and forth. And a certain iterative algorithm will make sure after many eight reasons,
the solution will converge. And sometimes the iterative process give you oscillating solution.
You need to do some regularization. But anyway, again, it's just the undergraduate level course.
So you notice the basic idea. So in summary, algebraic approach goes in following steps. So
first, you convert data into line integrals to form a system of linear equation. This is the first
step of a linear system. Solve the system of linear equations to reconstruct the underlying image
like iterative process I showed you. If needed, I didn't explain, but you need to regularize the
image reconstruction with prior knowledge. For example, you know CTE attenuation coefficient,
mu, means your attenuated x-ray intensity. That cannot be negative. So in the iterative process,
you correct the current solution. It give you negative one. You know it cannot be negative one.
You force negative thing to zero. That's a way to utilize prior knowledge to regularize the image
reconstruction. So you go iteratively refine an intermediate image or current guess one cycle by
one cycle until the outcome is satisfactory. How you know the outcome is satisfactory,
one way you look at image, make sense, giving you prior knowledge. Another way you see after
your correction, based on your current image, you predict the projection data. If your prediction
compare well with measurement, very close, and you say, okay, it's good enough. As long as
data fitness is concerned, we are doing good job. So this is the idea about the first algebraic
approach. The first one is not very hard, but at least you are sure that we can do so. Next one,
analytic approach involves Fourier slice theorem. That's a key point. So we need the Fourier analysis.
So here, if you already understand the Fourier analysis very well, you will have a good time here.
But if you're still confused about the Fourier analysis, you will feel a little struggling here,
but I think you need a review. This is not an easy thing, but anyway, it's very cool stuff.
Fourier analysis approach or analytic approach, let me give you a heuristic explanation.
We say from this point of view, you suit X-ray and you are not trying to get a reason.
This is different perspective. We say you suit X-ray, parallel beam X-ray, going this way,
going that way, and then you get line integral measurement. And what such measurement do?
And it's something I call it a probing wave. And because in this case, I think the underlying image,
you really shouldn't just think the image is Albert Einstein or your cross section of your chest.

It's just the image that you just mentally think. The image is really a summation of many waves.
This is because of Fourier analysis. You can always do so. You just think what you are going
to reconstruct is a bunch of waves. For example, you have waves propagating horizontally,
like this green wave. For this green wave and for any wave going horizontally,
let me make some comments so you understand why Fourier analysis can work nicely.
For such a horizontal wave, for all the X-ray projections along an oblique direction,
it's not a vertical direction. It's just an oblique or horizontal. But every orientation
except vertical projection, we can say one thing. You see, you do this vertical integral,
line integral, so you do vertical integral. The wave goes, you have a positive cycle,
you have a negative cycle, they cancel out, right? So the wave just keeps doing this.
You see, if the projection orientation is horizontal or makes an oblique angle
any degree except vertical, all these re-sums will give you zero.
Giving you zero means you get no information about an underlying wave, but only one direction,
vertical direction. So you go this way, so the positive cycle and the negative cycle
wouldn't cancel out. So you do vertical projection. Vertical projection, only
vertical projection, gives you critical information about a horizontal wave.
Okay, other directions really just cancel out. So from vertical projection, you get information
of wave propagation along the horizontal direction. And the Fourier analysis says the
image decomposed into many, many waves along different orientations. So to get horizontal
information, horizontal waves, you want to resolve horizontal waves, you use vertical
horizontal waves, vertical projection. So you got to say one degree,
one degree orientation. Those waves, you need projection, vertical 90 degree plus one
one degree. So just you need to think, if you want to resolve all the waves, you need
projection angle going through zero to 180 degree. So this is heuristics. So you need
orientation from different waves. And then you have the horizontal wave superimposed
this way, and then you got a vertical projection. So if you do Fourier analysis
along the horizontal direction, you should be able to recover all the wave information
along horizontal direction. So let me explain a little better. In so-called Fourier slice
theorem, it basically says this, okay, you'll have the parallel beam projection. So p of p
theta, this angle is theta. So you got all these projections. And this projection only carries
wave information along this direction. Okay, so this wave, this x-ray direction. So the
information carried by this projection profile will be wave along p-axis. This p-axis makes
angle theta. So it's the same thing. So see, this vertical direction carries wave information along
horizontal direction. So this projection at angle theta carries wave information along
p-axis, making this angle theta. So if you perform one-dimensional Fourier analysis,
okay, then you get the Fourier spectrum. This green profile is a Fourier

spectrum of the
right projection profile. So the Fourier spectrum will make, will be along the row axis, making theta angle, the same theta angle. And this point on this row axis gives you a wave at the frequency proportional to the distance between this
point to the system origin. It's a wave propagating along this theta direction orthogonal to the x-ray
beam direction. And along this row you'll have many, many points. These points represent a unique
two-dimensional wave propagating along this row direction, making angle theta. So this is
heuristics. And you need a one-dimensional Fourier transform to recover a radial line profile in
Fourier space. And to reconstruct two-dimensional images, two-dimensional image f of x, y,
you need to have all the Fourier information. So the theta angle need to go from 0 to 180 degrees.
So when the row axis changes from theta equal to 0, then 1, 2, 3, until 180 degrees,
so this whole Fourier space will be sweeped by the row axis. That means all the information,
if it's just one projection, you only measure information along this one line. But if you
change the theta from 0 to 180 degrees, so all the data points in the Fourier space
has been measured. You got all the information. Then you can perform two-dimensional
inverse Fourier transform. So this is a geometrical perspective on how you can reconstruct the image using Fourier analysis or from a perspective of wave analysis. So just
these few things, two slides. And I hope you understand these two slides.
Then we have 10 minutes rest. I will show you my thematics. So step by step,
you understand this geometrical wave argument a little better. OK.
I think the algebraic perspective may be easier to you. The wave analysis, if you feel confused,
feel free to ask me. Think about it. So this is, I think, a very elegant way to solve the problem.
The Fourier analysis is an important theorem for tomographic reconstruction.
You divide the error into three and three. Put three and three here. After that, you add these
two numbers together. The vertical projection will be, vertical resum will be six.
Then it will be same as your real measurement here. At this point, you just know error is six.
But you really don't know how you should redistribute it. You could put one here,
five here, or five here, one here. So what was your question?
Because you have no information. Just like a situation like this. You have two people,
say we together need $10. Then just all information you know. Then you have $10 to give them,
give the team. Then you need to give to each of them. So you just break into half, five, five.
You lack maximum entropy principle. You have no other information. So rather do it uniformly.
This is just the fire consideration. But mathematically we can prove such a way, under good condition, they can converge.
I'm not sure if you're going to fall, but I'm sure you're going to have space.
I know that it's like, it's easy to get confused, but not everybody can choose to get confused.
This is the only thing that we can help.
Like, we don't do this year, but we'll do next semester.
Thank you.
Thank you.

The whole aspect, when we look at the things that we can say,
this is where we're from, this is where we're from.
We're watching out for the environment.
And it's like, yes, well, no, yes, it's not that.
Why is it that?
When we're talking about people,
all those are just like,
people who,
yeah,
the people that are on the staff,
the people that are on the team,
all of us.
Yeah.
I'm probably just going to take five minutes,
so I'll see if someone has something to say.
Yeah.
I'm just going to ask a few questions, I think.
Yeah, I think that's a good idea.
I think that's a good idea.
I think that's a good idea.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Yeah.
Okay.
Okay.
So these two slides,
there's probing wave
and Fourier slice
theorem.
So I tried to use
these two slides to give you
ideas. So if you understand
the fundamental idea,
heuristically,
then the rest will be
just mathematical detail.
So I
uploaded a chapter by
Clark in a city book.
That's a very good one.
I was a fourth year
graduate student,
and I did a summer job
with my supervisor.
So in the summer, I read
through his book and implemented
the cone beam reconstruction
algorithm.
So this is kind of
the book, but still
the best engineering
book to explain

city principle. So the
green textbook,
you have explained
the back projection, but not
as clear as
Clark's book. So no heuristic
idea will
explain there, but
we try to just give
a better,
deeper explanation
utilizing
Clark's chapter 3.
So this is the code
in the system, and the
x, y, and you
just draw a line,
making an angle theta. You call this line
t-axis.
So the
projection will be done.
So perpendicular to this
t-axis, the distance
from a given
x-ray
to the origin.
So the line
goes through the origin. So this is
t. So it can be
x-price as
x-cosine plus y-sine
equal to t.
And here, so
the unit
directional vector
for this t-axis
is cosine
theta, sine theta. So this
is just along this direction.
So the unit vector
for x-coordinator
is cosine theta. For y-coordinator
is sine theta.
So the unit vector. They didn't draw
whether you think sine,
cosine theta, sine theta
are the two components
for the unit vector along
the t-direction.
Now arbitrary point, x, y
is arbitrary point. See on this
line, x, y.
So x, y, point
x, y to coordinate system
is a vector.
This vector
projected onto the
t-axis. So
what's the distance? That's
the distance t. So this
vector
determined by x, y
projected onto
unit vector

along the t-axis
with the components
cosine theta, sine theta. So this
is really inner product.
The two vector with
one with components, x, y,
the other with components, cosine
theta, sine theta. So the
left-hand side is inner product.
Inner product give you the
projected distance is t.
As long as t is the same,
so all the point
satisfy this equation.
It just mean the point
along this
straight ray with this
distance t.
This
specifies this
particular line. You do
line integral along the theta
t line. T gives this
distance. So this is
projection
t, theta, t.
Two variables, but for given theta
this is one-dimensional
function
of variable
t. So this is line integral.
This line integral
can also be
represented as a double
integral. The double integral you have
the underlying image.
Then along this
line you put the delta
function. So this is
not a point.
It's really
seat, the delta seat along
this direction. You do integral
and what this double
integral do is nothing
but this line integral.
So just do the
line integral along this
direction. So you have this
two-dimensional notation.
Then we can just write
standard Fourier transform
for this underlying
function f of x, y.
Equation
seven is two-dimensional
Fourier transform
straightforward. Let me write
one-dimensional Fourier transform
just the definition
straightforward equation eight.
So I have the
one-dimensional
function which is

projection profile p, theta
of t. So just one-dimensional
variable. Then you perform
Fourier transform. So you
got one-dimensional Fourier
transform with frequency variable
is w.
Here the two-dimensional
frequency variables u
and v. So these two
are just definitions
of two-dimensional
and one-dimensional
Fourier transform.
Then let's do a little bit
further. Consider
simplest
example, simplest case
and we try to compute
Fourier
analysis. Fourier analysis
is a two-dimensional
function of u, v. But we
set v value
to zero. That means
we only consider Fourier coefficient
along the u-axis.
That's just
one line through the
two-dimensional Fourier
spectrum. Given v
equal to zero, this becomes
one-dimensional function.
So you have this equation
seven becomes equation
nine, equation nine.
So the v equal to zero,
you got this part.
Just rearrange this a little bit
because now
this kernel does not
depend on, I mean,
this
exponential function
does not depend on
y. So the
integral with respect to y
can be grouped into
this inner integral.
So you got this part.
And you see this
expression in the bracket
is nothing
but a vertical integral.
Because you have two-dimensional function
you just do line integral
along the y-axis.
So this part
in bracket is nothing
but a vertical
integral.
So this is
nothing but a vertical integral.
Vertical integral

is projection profile
one theta equal to
zero. So that's just a vertical
integral. You got this one.
So this
Fourier analysis,
two-dimensional Fourier analysis,
f of u zero,
f of u zero,
so this vertical integral
is p theta equal
to zero x.
And just put
this part into
this
bracket.
Then you have
this expression.
Okay.
So this is two-dimensional
Fourier analysis along the
u-axis.
When v equal to zero.
So this is the u-axis.
And here is the vertical
integral. Vertical integral
is function of x
in the xy plane.
And this part is
one-dimensional Fourier analysis
with respect to
the variable x.
So this is one-dimensional
Fourier transform.
And you can put this
away. And that is
one-dimensional Fourier transform
one theta equal to
zero. And the variable
is u.
Okay.
So this is one-dimensional Fourier transform
in general
case for
arbitrary angle theta.
So this is a special case
of what I called
Fourier slice
theorem. So if you have
a vertical projection profile,
you perform one-dimensional
Fourier analysis, you get
a one-dimensional
Fourier spectrum.
You got one-dimensional
Fourier spectrum.
It's nothing but
the projection profile
in two-dimensional Fourier
transform space along
the u-axis.
So this is a special case
of Fourier slice
theorem. So when theta

is equal to zero.
So theta equal to zero, you got
two-dimensional Fourier
spectrum profile
along the one-dimensional
profile along the u-axis.
And how you get this
profile just along this
single line, u-axis,
special case.
You just make theta equal to
zero. You got vertical integrals.
Vertical integrals give you
one-dimensional signal.
You perform Fourier transform.
This is one-dimensional
Fourier transform.
And you get, you perform
so you do physical measurement
you got this profile.
You perform one-dimensional Fourier transform.
You got projection
profile along this line. And the
case I just show you, this line
is for theta equal to
zero along axis u.
Okay.
So this way you get
Fourier space information
recovered. This is by
only one line. But you got
extreme measurement and you
recover Fourier domain information.
Ideally you want
all the Fourier information
recovered.
Then you perform two-dimensional
inverse Fourier
transform. You will be
able to reconstruct underlying
image f of
x, y. And the
Fourier slice theorem is not
limited to this particular
theta equal to zero.
The theorem claims
for arbitrary theta.
This holds true. So you have
arbitrary theta. You got
arbitrary one-dimensional
projection profile.
And you perform one-dimensional
Fourier transform. You recover
the profile along this line.
This is what I explained in the
previous lecture. You keep
theta changing
from
zero to 180.
Then this line will sweep
whole Fourier space.
You got all information
recovered. So this is
the idea.

Actually, again, the heuristics
is that if you
have this vertical projection
property proved as such
and you can immediately
understand this must be
true. Why
I say so? Because Fourier
two-dimensional Fourier transform
has rotation property.
I mentioned you have object,
you have Fourier spectrum. You rotate
the object by 30 degrees.
What will happen to the
two-dimensional Fourier spectrum?
The Fourier spectrum should be also
rotated by 30 degrees.
So since
this theta angle
because of the
Fourier
transform rotation property,
this theta angle
is arbitrary. You can
select the angle as you set up
the system. So you can ask
what is, say, you
have this projection profile. You perform
one-dimensional Fourier transform, you
got this profile.
Will they be the same? They must be the
same because I could
select
my x-axis
along this direction.
My y-axis perpendicular
to that. In this
rotated new x-y coordinate,
I can just use what I explained
here. So the
Fourier slice theorem
must be true. So this is just
something you can immediately
understand based on
the rotation
property of two-dimensional
Fourier transform. But we can still
do so mathematically. So we
introduce rotated
coordinate system
t, s.
So this u, v, we
introduce this t, s.
And in, actually
this t, s is better
put in the x, y
space.
So you have this t, s.
I need to move this to
x, y place.
Then x, y plane.
So you can link
t, s coordinates to
x, y through this

straightforward coordinate
transformation.
Then you can just
go through mathematical
derivations to show
Fourier slice
theorem in general case.
This theta angle is the same
in the u, v domain
and in the x, y domain.
So you consider
the coordinate
transform. Then
p theta t
for angle theta
as a function of t
can be expressed
as a vertical
projection in the t, s
coordinate system.
So this is just the t, s. This is
s in this vertical
direction. But in the t, s
coordinate system. Then you
perform Fourier transformation
same thing. The one-dimensional
Fourier transformation. So you have
exponential part
e to the power minus
j two pi
w t. You got this one.
Then you do the same trick.
You just do
the
insert the definition
of the projection
profile for angle
theta. Insert
into bracket. You got this.
Pretty much like the simple case.
Simplest case I did.
Then you just
change
this transform back
from t, s to x, y
coordinate system.
So you got x-price in this
way.
You just rearrange a little bit
so this can be
x-price as a
two-dimensional Fourier transform
capital F.
Your component is w
cosine.
And the v component is w
sine theta.
Because you see here
the two-dimensional Fourier
transform really
just x u
plus y v
and w redistributed back.
So you got this

relationship. This is nothing
but the general form
of Fourier slice
theorem.
This is mathematical derivation.
You have time to review yourself
and you can check
this in chapter 3
if needed.
Let me just
visualize what we mentioned.
You have
underlying image
f of x, y
and use x-ray measurement.
You got
projection profile
p of theta t.
And for given theta
you perform one-dimensional
Fourier transformation.
Given theta
you perform one-dimensional
Fourier transformation with respect
to t. You got one line
profile
making the angle theta
in the two-dimensional Fourier
space. But theta can
be changed as you
wish from 0 to
pi, 180 degree.
So you got a line
this theta equal to 0.
This theta equal to
15 degree. For example
this theta equal to
80 degree. You have all
these theta. And this theta
equal to 170
degree for example. When
theta changes from
0 to 180 degree
so all these right radial lines
will fully cover
the Fourier space
u, v. So all the
values are measured
this way.
You know
f of u, v completely
you perform inverse
Fourier transform. Then you
recover f of
x, y. So
analytic process here
is completed
using Fourier transform
and particularly in
the form of Fourier
slice theorem. So this
is a Fourier imaging
example. So we
explain measured

data in the Fourier
space. So we try to
fit in the Fourier space
completely.
Then we can perform image
reconstruction. So this is just
a little bit
more
specialized
idea how you use
Fourier transform to do
CT reconstruction. Any questions?
Any questions?
Any questions?
Okay.
So let me go
a step deeper. So this
general idea. First I explain
very general idea. Then I
explain the Fourier slice
theorem. So little bit more specific.
Now go even
deeper. Give you specific
algorithm called
filtered back projection.
So this is just
nothing but
inverse Fourier transformation.
Not in the
rectangular coordinate,
but in the polar coordinate
because we keep changing theta.
So we better represent
the inverse
Fourier transform in the
polar coordinate system
so that what you measure
in polar coordinate system
you put into the formula.
So just the mathematical steps
let me go through.
So this is
inverse 2D
Fourier transform.
So if you know the Fourier spectrum
2D spectrum
capital F of u, v
you perform inverse transformation
you get F of x, y.
And my motivation is
to use
polar coordinate. So you
think of polar coordinate
you have radial
line in the Fourier space.
That's a row. Here
we call it W.
This is really the row I showed you before.
Polar you have polar
angle theta. So u
is nothing but
this
radius
W times cosine

theta. And v is
W times multiplied
sine theta.
The polar to rectangular
coordinate transformation.
And for this
d, u, d, v
in polar coordinate
d, u, d, v is a small
differential area element.
In polar coordinate you need
to have W, d,
W, d, theta.
So this small area
element is d, u,
d, v.
In polar coordinate
this line is
W.
Then you need
to say
this small angle
is d,
theta here.
So d,
theta times W
give you this arc
line.
W, d,
theta. W is radius.
And d, theta
give you this arc line.
Then you need d, W.
D, W give you this
increment.
So these times together is a small
area component.
Just your calculus stuff.
You see W, d,
W, d, theta.
So you put
inverse Fourier transform
in terms of
polar coordinate
system.
In polar coordinate
system, the polar angle
need to go full circle.
And radius go from 0
at system origin
all the way to infinity.
So you cover the full space.
Then we do a little trick here.
We decompose
this full circle
into two half-scan
from 0 to pi.
Here from pi to 2 pi.
But it's still 0 to pi.
Because I put theta
plus, instead of
just theta, I put theta plus
pi here. So theta plus pi, theta
plus pi. That's why

this pi to 2 pi
becomes 0 to
pi. So you got this part.
For parallel beam geometry,
so the angular range
from 0 to 180
degrees is enough.
So if you have
0 to
360 degrees,
so you double the information.
You really just need half of it.
So we can
mathematically deal with
the problem.
And by changing
this
capital F
W theta plus
180 degrees here,
just
change back to
theta.
Here we want to change back to theta,
but cosine theta plus
pi equal to
minus cosine theta.
Here is minus sine theta.
So taking all these trivial
transformations, here
is the property
we know. We can easily
verify for Fourier
analysis.
You have 180 degrees,
then you just need your same angle,
but you just reverse sine
of omega. So with all
these things, you have
0 to pi, because you only
need this 0 to pi
here.
And then you have
this part.
You really
change
W
to minus W. So this
0 to infinity
becomes
0 to minus
infinity. Then we just
change the limit. So we
put things together.
You have this F of x, y
equal to double integral.
The inner integral is
from minus
infinity to positive
infinity.
So all these are just
mathematical details. If you
got lost, you review.
You ought to be able to follow.

But anyway, now we end
up with this formula.
This formula is what we
call the filtered back
projection. So if you got
lost somewhere, it's not
critical. You can review. But
you just trust me for now.
And through this step,
you got this one. This is
filtered back projection.
Why I call it
filtered or why
we have the filtration?
You see this
this is
here. It's a
Fourier spectrum along the
radial line. So the
Fourier spectrum along
that direction, if you just
perform
if you just perform
inverse Fourier
transformation, what do you have?
You will have original
projection profile because
this is
one-dimensional
Fourier transform
yeah
one-dimensional Fourier transform
or projection profile. If you
do not have this
absolute value w
you just perform inverse Fourier
transform, you go back
to what you measured
one-dimensional projection profile.
But here you have
this additional
factor. That means
if you put these two things
together, that means original
Fourier spectrum
has been modified
by absolute value w.
So w is small
when you close to system origin.
So for low-frequency
components,
this vector will be small.
For high-frequency
components, w, absolute
value w is larger.
So the higher-frequency
the larger w value
as a weighting factor
weighting factor
upon the high-frequency
components of
this one-dimensional
Fourier spectrum. Therefore
this product

so S w times
absolute value w
is high-pass
filtering. So the Fourier
spectrum is modified with
high-frequency components
elevated according to absolute
value w. So after
the spectral modification
you perform
inverse Fourier transform
you go back to the projection
domain. Due to the
high-frequency
enhancement
this inverse Fourier
transform is
no longer
original projection profile.
It's rather a modified high-pass
filtered
projection profile.
It's not P theta of T
P theta of T anymore
it's called Q theta of T
it's a filtered projection
profile. This is why I call
it filtered.
What do I mean
by back projection?
So you do filtration first
this one-dimensional filtration
this filtered
projection profile
all the values
are put back into
field of view. According to
this argument
so any value x, y
you just retrieve
certain value from your filtered
projection profile.
You put that value back
so this is back projection
process. So I say
this is a filtration
that may be a little
clear
but when I say this is back projection
maybe you feel a little
confusing. So here
we have this picture
the back projection is visualized
so this is
the projection profile
after filtration
so it is filtered
projection profile.
This filtered
projection profile
Q theta I of T
is really
smeared back over the field of view
so you think this

projection profile
is just smeared back over the field
of view from this
particular direction. So you have
many filtered projection profiles
you sweep all of
you smear all of them back
over field of view.
Add it together, give you results.
So see
for given x, y
what's the contribution
from given Q theta?
So this is a question
I want to explain. So given
x, y say that
x, y is here for example
for this given x, y
the point in
the image space
how much contribution we can get
from a particular
filtered
projection profile
here is Q theta I
so you need to do this
so for this x, y
for given theta I
you do this inner product
what you get is T
T is distance
between this line
and this central line
and both of them are
perpendicular to this
by making angle theta I
so for any point
along this line
this inner product
as I explained
will be the same T
so as long as the pixel
is on this line
with distance T
and you will
need this value
to retrieve
this filtered
projection profile
so for all point on this line
this T will be the same
so this T
is computed here
so you get this value
so this value will be put back
for any x, y
as long as x, y
the point x, y is on this line
so this value is
same here, here, here, here
is all the way same
so that value if it is one
so along that line every point
is put in one

so this is what I call the smearing back
so this is just for this
theta, theta I
and for other one
you just smear back
from different way
and the process is linear
because the integral means summation
so for each theta
you retrieve
the value
from Q theta
then add it back
to the particular point
x, y and for another angle
you get the value similarly
so just think of this image
so from this you got this
from
this filtered projection
profile you get value here
but if you say
along this direction you have
a projection profile and you will
get the value again
so through the particular
point x, y going
through back this way
so you can view it for this
particular thing
the pixel is here
for this view you see where you get value
but for
a projection profile
this way you will get central
value
so whatsoever you have
the heuristic picture
really you can
think from
filtered projection domain
any profile
you smear back uniformly
so the value
cover all the point
all the different projection
angles added back
the same way
then you have the
image value recovered
so think about that
I will show you in minutes
some reconstructing
examples you can
have a better idea
ok
now several slides
with green buttons
about reconstruction
filter
here reconstruction filter
high pass filter
is absolute value w
and this filter

this filter does not
have inverse Fourier
transform because
absolute value w is
not integrable
however we can introduce
bound limitedness
assumption
for given projection profile
you have a maximum
frequency
maximum bandwidth
capital w
so
the filtration
really need to be done
with this truncated
filter
so the original
capital in Fourier
space
this is absolute value
of omega
so this goes on infinitely
then you assume you have
maximum bandwidth
capital w
so anything outside
the only meaningful thing
is this part
so this part becomes
integrable
so you use this window
you truncate
otherwise divergent
high pass filter
filtering kernel
then you perform inverse
transform with respect to
this truncated version
h of omega
you do inverse Fourier transform
so you can compute
the inverse Fourier
transform as the
spatial domain
counterpart of high pass filter
so you do some computing
you got this part
now we assume
capital w
is maximum bandwidth
and because of that
we can use the sampling
theorem
so the projection profile
can be expressed
through the sample
kernel
in terms of
discrete sampling
point p theta k tau
tau is the sampling step
and likewise

this filtering kernel
under assumption
of limited bandwidth
capital w can be
expressed in terms of
sampled value
so you got these two equations
so the continuous domain
expression
can be expressed
in terms of discrete data point
so you got
the filtered
projection profile
in terms of
sampled data point
so this is the band
limited high pass filtering
so you can
read if you are interested
but anyway
just some practical
implementation detail
about high pass
high pass filtering
for
filtered bag projection
so what I have explained
is only
the
two dimensional case
and in three dimensional case
you have an extended
higher dimensional version
of Fourier slice theorem
and the mathematics
will be a little more
complicated but the essential
idea is still the same
so you have a 3d object
you view it as superposition
of 3d waves
it's not 2d waves
all kinds of orientation
frequency then from one direction
superimposed together
you only get information
for the
for the wave components
with the
directional
vector and the frequency
component
represented in plane
orthogonal to the
projection direction
so this is really just
extension of
the 2d Fourier slice
theorem I don't want to
confuse you too much again you see
this nice
green button
so analytical

pros we convert
x-ray data into
Fourier space
also we could
equivalently put in
redone space but now
only explain the Fourier space
processing
and inward Fourier
space transform
according to
close the form formula
like I derived
filter back projection
is analytical formula
you do not need to do
iterative reconstruction
so in the
reconstruction process
some filtering prior
processing steps may be used
and iterative algorithm
is very
useful when
the data are not complete
say you miss some data
are blocked by metal
or there are
some other imperfectness
in data acquisition process
analytical pros you don't
need iterative
process you have formula that's nice
but it assumes no
noise and the data is
complete so
the 2 pros is
iterative and analytical
have their strengths
and weaknesses
and nowadays
iterative process
algorithms are more popular
because we deal with
low dose reconstruction
very short data acquisition
time
so
filtered back projection
as I explained here
is just a mathematical
formulas and some geometrical
pictures I mentioned
now I'm going to show you
last 10 minutes
show you some numerical examples
so you have better understanding
these are
examples you see how
what we learned will work
with real example
so the real example
is simple
cross-section with a

small bright
disk here
you do
one dimension
one dimensional projection
data acquisition
with angle theta
you got this profile
you got this peak due to this
bright disk
so this
is at angle theta theta can be
changed from 0
degree to pi
so you got this way
it's tracing half
of the sinusoidal curl
and for given projection profile
is here
so we have
example but you have
at different elevation
you have different theta angle
so you have different data
so if you do back projection
what will happen
I explained a lot about
back projection
back projection is
the filtered projection profile
you smear it back
over field of view
so let me show you
what happened
let's just don't bother to do any
filtration
this is the projection profile
I do back projection
so this is projection profile
something like this
you do back projection
you smear the profile back
along the original
x-ray path
back over the field of view
so the higher value here
is put back
so along this line
the value is
somewhere here
so this value is put back
every pixel
along this line
here the value is a little low
so you see along this line
every point is a little dark
so you just find the pixel
in pixel on that line
you take a value
and you smear back
this is from one direction
and you do so from every direction
the smear
back results

are added together
because the integral is
with respect to d theta
integral is just summation
so you smear back one projection
you get a picture like this
you get another projection
you get another pixel
maybe say the projection profile is here
you smear the back picture
the other way
you add two things together
many projections add many things together
the results you have
from 180 degrees
you add back everything together
and something
not too much
different from the truth
but it looks very much blurred
you still see this
is a little high
because
see
this part is high
you back project back
so this value along this line is high
so the back projection
is along the original
actually path
so the high value will be put high
but there are some
traits
say when the
smearing back
in the smearing back process
here you got high value
there are no high value there
just the back projection
process will do so
so you got the smear back
all the high value around
so simply you collect
projection
then you do back projection
only is not enough
to recover accurate
image that's why
we really need to filter the projection
so you have
projection profile you do not
just smear back
you really first do filtration
high pass filtration
with kernel H
so you do filtration
so you got this
high pass filtered
projection profile called
P prime
so you got this one
then you do back projection
so this is filtered back projection
and you do back projection

for every filtered
projection profile
so this is synogram
and filtration is one dimensional
horizontal
along horizontal direction
so high pass filtering means
the edge
will be
emphasized
so this is original synogram
this filtered
synogram something like this
so once filtered
so this filtered data
you do
back projection
then you can recover
the original image
this look pretty much
the original image
so if you have many angles
the angular increment is very small
detector spacing
is very small
this becomes closer and closer
to the true image
so this is just
numerical example
next few slides about
MATLAB
implementation
also your homework
so you can just
read through
and try out yourself
so MATLAB
has
command tool box
implement
projection, filtration
filtered back projection
so the keyword is
so you have image
theta can be a single value
that just give you one projection
can be a ray that give you
many projection
then the inverse Fourier transformation
will be done
with eye radon
so the beam
projection beam line by line
so line spacing is
one pixel
one pixel unit apart
shown something like here
when you do projection
they basically decompose
a single pixel
into two by two
matrix
then you shoot rays through all this center
and in the projection

domain
so you see if this ray hit
the center
and the detector take all the value
if it hit the boundary between
two detector
the value will be equally divided
so if it hit somewhere
arbitrary location
the linear interpolation will be used
so this
sample read on code
you can run yourself
and you generate
image with this white
block and you generate
projection data
you do filtration
this red arrow means pictures
are in the
on next slide
so you got this projection profile
then you use
inverse read on code
essentially you do filter
back projection
you can
do linear interpolation
and that means
without filtration
just without filtering
just do simple back projection
you see unfiltered
back projection
this filtered back projection
pretty much original
image
so this is the secret
how you can reconstruct
CT images
and show you some clinical examples
I think that's magic
and by now you do your review
and if you didn't do
preview
and you do not do review
I assure you no one would understand
all the trick
but if you just read carefully
you will be able to understand the secret
how the X-ray machine you're laying down
and just the X-ray going through
you'll never see
but the internal structure
clearly revealed
and you see any problem
down to the resolution
1 third millimeter
every detail laid out
using the algorithms
I explained to you
so this is something
I think very amazing
achieve inner vision

with X-ray homework
just review what we have
another example
I show you with a block
and you try to do same thing
but you generate ellipse
and just go through
and the MATLAB code
and some space variables
you may not immediately
understand looking at slides
but you just click
Google MATLAB help
and everything is straightforward
you just need to spend some time
and get familiar
see how filter
the bag projection will work
for you
so much for today
thank you